Gravity From a Buried Sphere

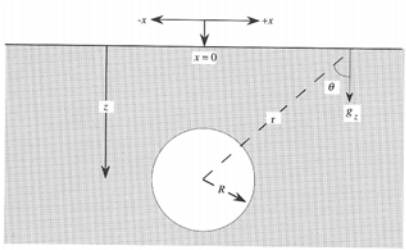


Figure 6-13 Notation used in derivation of the gravity effect of a buried sphere. The same notation is used for a traverse at right angle to the strike of a horizontal, infinitely long cylinder.

Gravity from a Sphere

excess mass due to the sphere is equal to $4\pi R^3\rho_c/3$. Using Eq. 6-4 we see that the gravitational attraction of the sphere in the direction of r is

$$g_{sphere(r)} = \frac{Gm}{r^2} = \frac{G4\pi R^3\rho_c}{3r^2} = \frac{G4\pi R^3\rho_c}{3\left(x^2 + z^2\right)} \qquad (6\text{-}33)$$

We seek the vertical component of $g_{sphere(r)}$ because gravimeters respond only to gravity in the vertical direction. In general, unless noted otherwise, the component in the vertical direction g_z is implied. Therefore,

$$g_{sphere} = \frac{G4\pi R^3\rho_c}{3\left(x^2 + z^2\right)}\cos\theta = \frac{G4\pi R^3\rho_c}{3}\frac{z}{\left(x^2 + z^2\right)^{3/2}} \qquad (6\text{-}34)$$

Now that we have the required equation, let's study some attributes of gravity over a buried sphere. Table 6-6 uses Eq. 6-34 and allows us to directly compare gravity over two spheres, so we can vary parameters in each and observe the differences. In case you are interested in making similar computations for other equations we present, Eq. 6-35 illustrates units conversion, so g is in mGal when density is in g/cm³, distances are in meters, and G is in dyne • cm²/g².

$$g(\text{mGal}) = 6.67 \times 10^{-8}\,\frac{\text{dyne}\cdot\text{cm}^2}{\text{g}^2}\cdot\frac{\text{g}}{\text{cm}^3}\cdot\frac{\text{m}}{1}\cdot\left(\frac{\text{g}\cdot\text{cm}/\text{s}^2}{\text{dyne}}\cdot\frac{100\text{ cm}}{\text{m}}\cdot\frac{\text{gal}}{\text{cm}/\text{s}^2}\cdot\frac{1000\,\text{mGal}}{\text{Gal}}\right) \qquad (6\text{-}35)$$

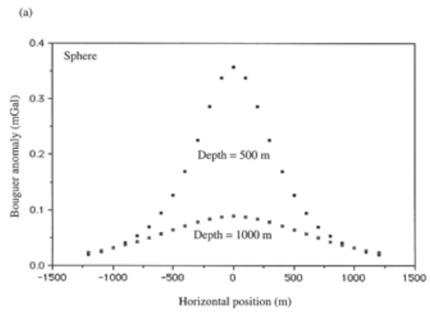
Gravity over a Sphere

TABLE 6-6 Gravity Over a Sphere

Sphere A		Sphere B	
Sphere radius (m)	**200**	Sphere radius (m)	**200**
Depth to sphere center (m)	**500**	Depth to sphere center (m)	**1000**
Density contrast (g/cm3)	**0.4**	Density contrast (g/cm3)	**0.4**
Horizontal increment	**100**	Horizontal increment	**100**

Horizontal Position (m)	Gravity (mGal)	Horizontal Position (m)	Gravity (mGal)
−1200	0.0203	−1200	0.0235
−1100	0.0253	−1100	0.0272
−1000	0.0320	−1000	0.0316
−900	0.0410	−900	0.0367
−800	0.0532	−800	0.0426
−700	0.0702	−700	0.0492
−600	0.0938	−600	0.0564
−500	0.1264	−500	0.0640
−400	0.1703	−400	0.0716
−300	0.2255	−300	0.0786
−200	0.2862	−200	0.0843
−100	0.3372	−100	0.0881
0	0.3576	0	0.0894
100	0.3372	100	0.0881
200	0.2862	200	0.0843
300	0.2255	300	0.0786
400	0.1703	400	0.0716
500	0.1264	500	0.0640
600	0.0938	600	0.0564
700	0.0702	700	0.0492
800	0.0532	800	0.0426
900	0.0410	900	0.0367
1000	0.0320	1000	0.0316
1100	0.0253	1100	0.0272
1200	0.0203	1200	0.0235

Gravity over a Sphere



Gravity Effect of a Horizontal Cylinder

Figure 6-13 equals πR^2, substituting into Eq. 6-20 gives us, for gravity in the direction of r,

$$g_{cylinder(r)} = \frac{G2\pi R^2 \rho_c}{r} \qquad (6\text{-}36)$$

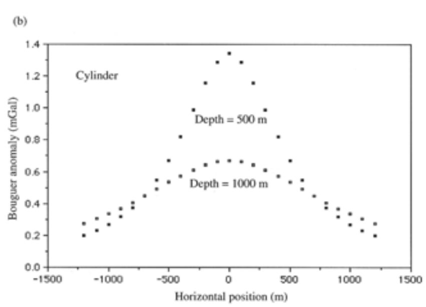
Thus, the equation for the vertical component becomes

$$g_{cylinder} = \frac{G2\pi R^2 \rho_c}{r}\cos\theta = \frac{G2\pi R^2 \rho_c z}{r^2}$$

and

$$g_{cylinder} = G2\pi R^2 \rho_c \frac{z}{\left(x^2 + z^2\right)} \qquad (6\text{-}37)$$

Gravity of a Horizontal Cylinder



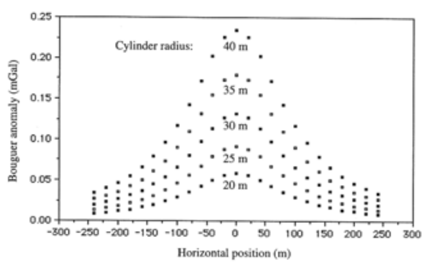
Gravity Over Horizontal Cylinder

TABLE 6-7 Gravity Over a Horizontal Cylinder

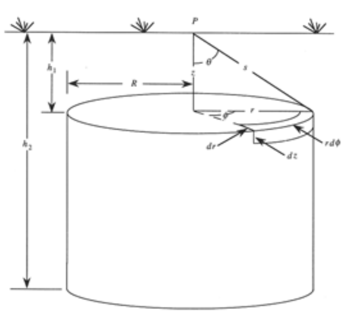
	Cylinder A		Cylinder B
Cylinder radius (m)	**200**	Cylinder radius (m)	**200**
Depth to cylinder center (m)	**500**	Depth to cylinder center (m)	**1000**
Density contrast (g/cm3)	**0.4**	Density contrast (g/cm3)	**0.4**
Horizontal increment	**100**	Horizontal increment	**100**

Horizontal Position (m)	Gravity (mGal)	Horizontal Position (m)	Gravity (mGal)
−1200	0.1984	−1200	0.2748
−1100	0.2296	−1100	0.3034
−1000	0.2682	−1000	0.3353
− 900	0.3163	− 900	0.3705
− 800	0.3767	− 800	0.4089
− 700	0.4531	− 700	0.4500
− 600	0.5496	− 600	0.4930
− 500	0.6705	− 500	0.5364
− 400	0.8177	− 400	0.5781
− 300	0.9861	− 300	0.6152
− 200	1.1561	− 200	0.6448
− 100	1.2895	− 100	0.6639
0	1.3411	0	0.6705
100	1.2895	100	0.6639
200	1.1561	200	0.6448
300	0.9861	300	0.6152
400	0.8177	400	0.5781
500	0.6705	500	0.5364
600	0.5496	600	0.4930
700	0.4531	700	0.4500
800	0.3767	800	0.4089
900	0.3163	900	0.3705
1000	0.2682	1000	0.3353
1100	0.2296	1100	0.3034
1200	0.1984	1200	0.2748

Horizontal Cylinder - Radius



Vertical Cylinder



Vertical Cylinder

$$\Delta g_{vertical\ cylinder(s)} = \frac{G\rho_c dz\,(r d\phi)\ dr}{s^2} \quad (6\text{-}38)$$

$$\Delta g_{vertical\ cylinder} = \frac{G\rho_c dz\,(r d\phi)\,dr}{s^2}\cos\theta = \frac{G\rho_c dz\ (r d\phi)\,dr}{s^3}\,z$$

and

$$\Delta g_{vertical\ cylinder} = \frac{G\rho_c\,(z dz)(r dr)d\phi}{\left(r^2 + z^2\right)^{3/2}} \quad (6\text{-}39)$$

Next we determine the gravitational attraction of the cylinder by using the volume element to sweep out a thin ring, then a thin disc, and finally the cylinder. Equation 6-40 gives the formula for this integration.

$$g_{vertical\ cylinder} = \int_{h_1}^{h_2} dz \int_0^R dr \int_0^{2\pi} \frac{G\rho_c\,(z)(r)}{\left(r^2 + z^2\right)^{3/2}}\,d\phi \quad (6\text{-}40)$$

The result of this integration is Eq. 6-41.

$$g_{vertical\ cylinder} = G\,2\,\pi\rho_c\left(h_2 - h_1 + \sqrt{R^2 + h_1^{\,2}} - \sqrt{R^2 + h_2^{\,2}}\right) \quad (6\text{-}41)$$

Note that this equation gives gravity only at a point above the cylinder axis and is not valid for determining gravity at points away from the axis.

Vertical Cylinder -> Terrain

There are some other interesting results that arise from this derivation. When evaluating the integral from 0 to R, if we assume R goes to infinity, the result is an equation for an infinite slab (the Bouguer correction). Hence, we could have used this approach to determine the Bouguer correction. In addition, if we have a large cylinder with radius R_o and a smaller cylinder (with radius R_i) with an axis coincident with the larger cylinder and with coincident top and bottom, we can determine the effect of a ring by subtracting the effect of the smaller cylinder from the larger. Let P be on the top of the cylinder, so that $h_1 = 0$, and let the bottom of the cylinder be a distance z from the top, then

$$g_{ring} = G2\pi\rho_c\left[\left(z + \sqrt{R^2_o} - \sqrt{R^2_o + z^2}\right) - \left(z + \sqrt{R^2_i} - \sqrt{R^2_i + z^2}\right)\right]$$

and

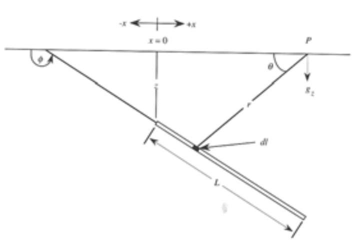
$$g_{ring} = G2\pi\rho_c\left(R_o - R_i - \sqrt{R^2_o + z^2} + \sqrt{R^2_i + z^2}\right). \quad (6\text{-}42)$$

Equation 6-42 is the terrain correction.

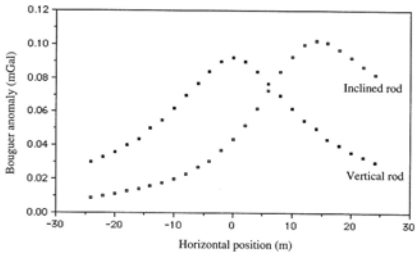
Inclined Rod

In the case of a vertical rod (inclination = 90°), the equation becomes less complex and is

$$g_{vertical\ rod} = G\rho_c\pi R^2\left\{\frac{1}{\left(z^2 + x^2\right)^{1/2}} - \frac{1}{\left[(z + L)^2 + x^2\right]^{1/2}}\right\} \quad (6\text{-}43)$$



Inclined Rod



Inclined Rod

TABLE 6-8 Gravity Over Inclined and Vertical Rods

Inclined Rod		Vertical Rod	
Rod radius (m)	**1**	Rod radius (m)	**1**
Depth to rod top (m)	**10**	Depth to rod top (m)	**10**
Rod length (m)	**100**	Rod length (m)	**100**
Rod inclination (degrees)	**135**	Rod inclination (degrees)	**********
Density contrast (g/cm3)	**0.4**	Density contrast (g/cm3)	**0.4**
Horizontal increment (m)	2	Horizontal increment (m)	2

Horizontal Position (m)	Gravity (mGal)	Horizontal Position (m)	Gravity (mGal)
−24	0.009	−24	0.030
−22	0.010	−22	0.033
−20	0.011	−20	0.036
−18	0.013	−18	0.040
−16	0.014	−16	0.044
−14	0.016	−14	0.050
−12	0.018	−12	0.055
−10	0.020	−10	0.062
− 8	0.023	− 8	0.070
− 6	0.027	− 6	0.077
− 4	0.031	− 4	0.084
− 2	0.037	− 2	0.090
0	0.044	0	0.092
2	0.052	2	0.090
4	0.062	4	0.084
6	0.073	6	0.077
8	0.084	8	0.070
10	0.093	10	0.062
12	0.100	12	0.055
14	0.102	14	0.050
16	0.101	16	0.044
18	0.097	18	0.040
20	0.092	20	0.036
22	0.087	22	0.033
24	0.082	24	0.030

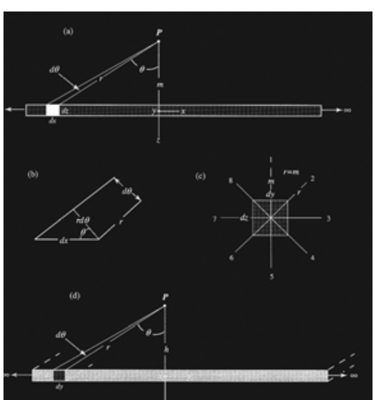


FIGURE 6.4 Relationships and notation used to derive the Bouguer correction.

Gravity Due to an Infinite Rod

The attraction of a thin rod is the sum of all these components, so we write

$$g=\int_{x=-\infty}^{x=\infty}\frac{G(\rho dx dy dz)}{r^2}\cos\theta . \qquad (6.18)$$

Before evaluating the integral, we note that $r = m/\cos\theta$. Also (see Figure 6.4(b)), because the length of arc for a small angle $d\theta$ equals the radius r times the change in angle $d\theta$, it is straightforward to demonstrate that $dx = r\,d\theta/\cos\theta$. Using these relationships, we show that

$$\frac{G(\rho dx dy dz)}{r^2}\cos\theta=\frac{G(\rho dy dz) r d\theta \cos\theta(\cos\theta)^2}{m^2\cos\theta}=\frac{G(\rho dy dz) m d\theta\cos\theta(\cos\theta)^2}{m^2\cos\theta\cos\theta}=\frac{G(\rho dy dz)}{m}\cos\theta\, d\theta .$$

Using this result, we adjust Equation 6.18 so that

$$g=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{G(\rho dy dz)}{m}\cos\theta\, d\theta \qquad (6.19)$$

which evaluates to

$$g=\frac{2G\rho dy dz}{m}. \qquad (6.20)$$

Equation 6.20 gives us g at P over a thin rod as illustrated in Figure 6.4(a).

Gravity Due to an Infinite Slab

The next step in our derivation is to sweep out a thin sheet using a thin rod (that is, use Equation 6.20 to calculate g over a thin sheet). Figure 6.4(c) is an end view of our thin rod. If lines 1, 2, 3, 4, 5, 6, 7, and 8 are of equal length, then g is the same at the end of every line. Line 1 is equivalent to m in our original diagram, and we will refer to line 2 as r. Clearly $m = r$ and g is the same at positions 1 and 2. This gives us the same view of the thin rod as we see in Figure 6.4(d), where line 2 now is labeled as r. Because we know g due to the thin rod (Equation 6.20), we use this to write

$$\Delta g_r=\frac{2G(\rho dy dz)}{r}. \qquad (6.21)$$

The next several steps mimic exactly those that we just listed, so following the same procedure we can say that

$$\Delta g_z=\frac{2G(\rho dy dz)}{r}\cos\theta \qquad (6.22)$$

$$g=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2G(\rho dz) d\theta \qquad (6.23)$$

and

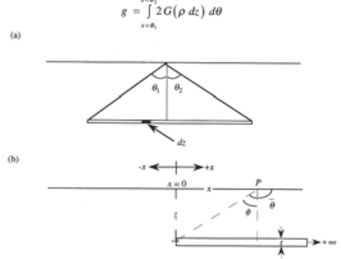
$$g = 2\pi G(\rho dz). \qquad (6.24)$$

Horizontal Sheet

Gravity Effect of a Horizontal Sheet

When deriving the Bouguer effect, we created a thin slab by moving a thin rod from +∞ to -∞ (integrating from $\pi/2$ to $-\pi/2$; see Eq. 6-23). If instead we integrate from θ_1 to θ_2 (see Fig. 6-19(a)), we have

$$g=\int_{x=\theta_1}^{x=\theta_2} 2G(\rho\, dz)\, d\theta \qquad (6\text{-}44)$$



Horizontal Sheet

and

$$g = 2G\rho_c\left(dz\,\overline{\theta}\right) \quad (6\text{-}45)$$

where $\overline{\theta}$ represents the included angle in radians. If we keep the slab thin with respect to its depth, we can write

$$g_{truncated\ slab} = 2G\rho_c t\left(\overline{\theta}\right) \quad (6\text{-}46)$$

where t is the thickness of the truncated slab (Fig. 6-19(a)). Our goal in this section is to derive a relationship for a semi-infinite slab (Fig. 6-19(b)). For this case the included angle $\overline{\theta}$ is equal to $\pi/2 + \tan^{-1}(x/z)$ and we therefore have

$$g_{semi-\infty\ sheet} = 2G\rho_c t\left(\frac{\pi}{2} + \tan^{-1}\frac{x}{z}\right) \quad (6\text{-}47)$$

This approximation is within 2 percent if $z \geq t$ (Telford et al. 1990, p. 40). Table 6-9 uses

Horizontal Sheet

TABLE 6-9 Gravity Over Semi-Infinite Sheets

Sheet A		Sheet B	
Depth to sheet (m)	4	Depth to sheet (m)	8
Sheet thickness (m)	1	Sheet thickness (m)	1
Density contrast (g/cm3)	0.4	Density contrast (g/cm3)	0.4
Horizontal increment (m)	2	Horizontal increment (m)	2

Horizontal Position (m)	Gravity (mGal)	Horizontal Position (m)	Gravity (mGal)
24	0.0009	−24	0.0150
22	0.0010	−22	0.0149
20	0.0011	−20	0.0147
18	0.0012	−18	0.0145
16	0.0013	−16	0.0143
14	0.0015	−14	0.0140
12	0.0017	−12	0.0136
10	0.0020	−10	0.0132
8	0.0025	−8	0.0126
6	0.0031	−6	0.0118
4	0.0042	−4	0.0109
2	0.0059	−2	0.0097
0	0.0084	0	0.0084
−2	0.0109	2	0.0071
−4	0.0126	4	0.0059
−6	0.0136	6	0.0049
−8	0.0143	8	0.0042
−10	0.0147	10	0.0036
−12	0.0150	12	0.0031
−14	0.0153	14	0.0028
−16	0.0155	16	0.0025
−18	0.0156	18	0.0022
−20	0.0157	20	0.0020
−22	0.0158	22	0.0019
−24	0.0159	24	0.0017

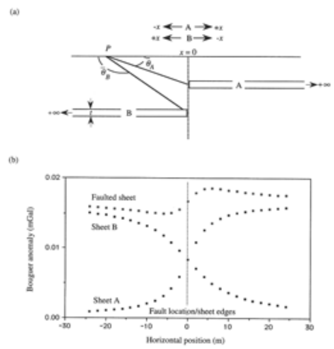
Faulted Horizontal Sheet

The equation for the faulted horizontal sheet is

$$g_{faulted\ sheet} = 2G\rho_c t\left[\pi + \tan^{-1}\left(\frac{x}{z_1} + \cot\phi\right) - \tan^{-1}\left(\frac{x}{z_2} + \cot\phi\right)\right] \quad (6\text{-}48)$$

where x is the horizontal position of the measurement location, t is sheet thickness, ϕ is fault inclination, z_1 is depth to upthrown block, and z_2 is depth to downthrown block. Note that the relative positions of the faulted sheet are constant (i.e., the upthrown block is always

Faulted Horizontal Sheet



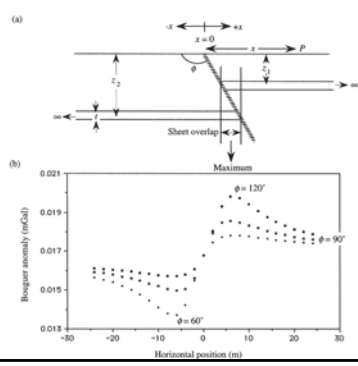
Faulted Horizontal Sheet

TABLE 6-10 Gravity Over a Faulted Horizontal Sheet

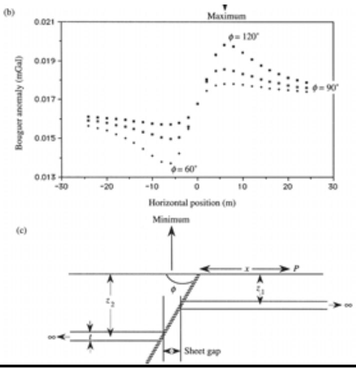
Depth to downthrown sheet (m)	8
Depth to upthrown sheet (m)	4
Sheet thickness (m)	1
Fault inclination (degrees)	90
Density contrast (g/cm3)	0.4
Horizontal increment (m)	2

Horizontal Position (m)	Gravity (mGal)
−24	0.0159
−22	0.0159
−20	0.0158
−18	0.0157
−16	0.0156
−14	0.0155
−12	0.0153
−10	0.0152
−8	0.0150
−6	0.0150
−4	0.0150
−2	0.0156
0	0.0168
2	0.0179
4	0.0185
6	0.0186
8	0.0185
10	0.0183
12	0.0182
14	0.0180
16	0.0179
18	0.0178
20	0.0177
22	0.0177
24	0.0176

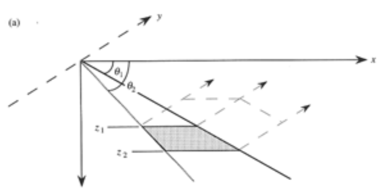
Reverse Fault



Normal Fault



Model Cell



Model Cell

lowing a procedure much like we employed to determine the Bouguer correction, the semi-infinite sheet, and the vertical cylinder. If we have a small cell of infinite length in the y-dimension with side lengths dz and $d\theta$, the gravity effect of such a cell is easily determined by integration. Thus, we have

$$g_{cell} = 2G\rho_c \int_{\theta_1}^{\theta_2} d\theta \int_{z_1}^{z_2} dz \qquad (6\text{-}49)$$

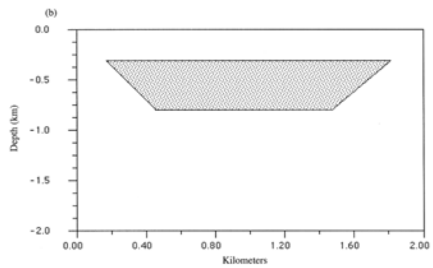
or

$$g_{cell} = 2G\rho_c(\theta_2 - \theta_1)(z_2 - z_1) \qquad (6\text{-}50)$$

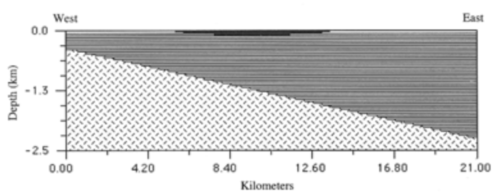
The notation is defined in Figure 6-22(a). All we need now do is to represent a polygon such as that illustrated by Figure 6-22(b) as an accumulation of such cells. If a polygon has n such cells, then the gravity effect of the polygon is

$$g_{polygon} = 2G\rho_c n\,\Delta\theta\,\Delta z \qquad (6\text{-}51)$$

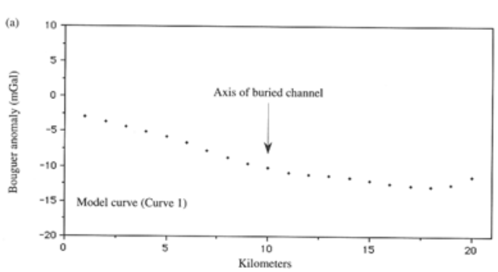
Polygon



Regional/Residual



Regional/Residual



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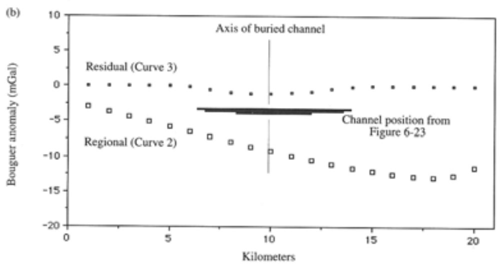
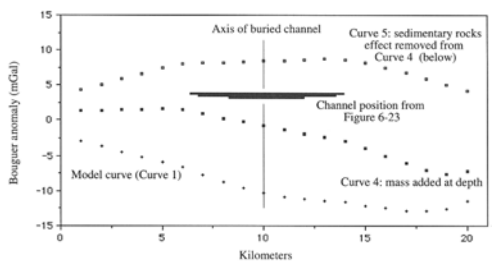
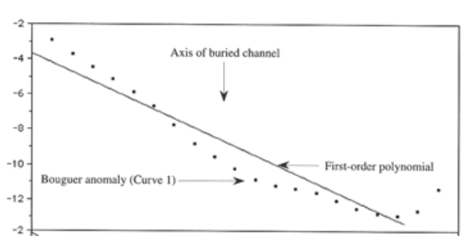


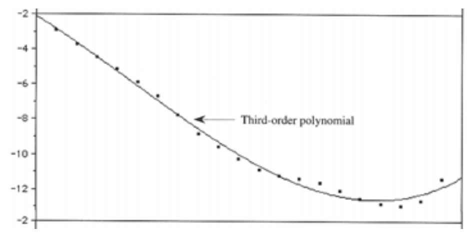
Regional/Residual



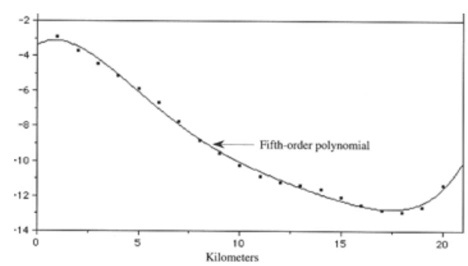
Regional/Residual



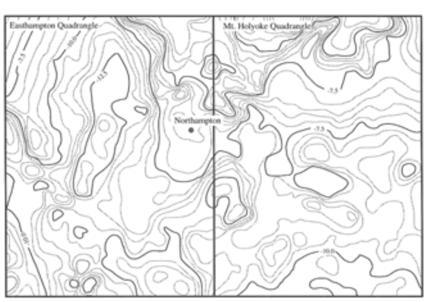
Regional/Residual



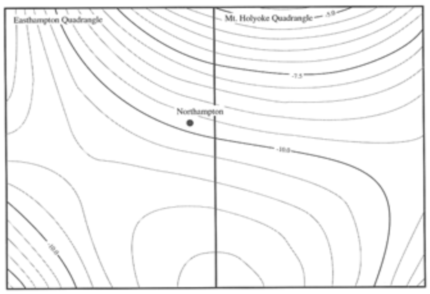
Regional/Residual



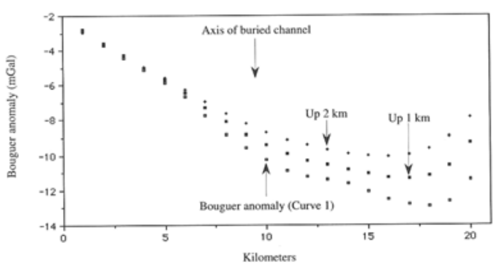
Regional/Residual



Regional/Residual



Upward Continuation



Upward Continuation